Marginal demand and elasticity with implicit function

Given the equation:

$$p_x + p_y + D_x - e^{p_x p_y} D_x = 0$$

which implicitly defines the demand for good x as a function of prices, $D_x = f(p_x, p_y)$, in the vicinity of the point $P_0 = (p_{x_0}, p_{y_0}, D_{x_0})$.

1. Find the marginal demand with respect to the price p_x at the point P_0. Classify the good and provide the economic interpretation of the result.
2. Find the elasticity of demand with respect to the price p_y at the point P_0, and interpret the result economically.

Solution

1. To find $\dfrac{\partial D_x}{\partial p_x}$, we use implicit differentiation:

$$\frac{\partial F}{\partial p_x} + \frac{\partial F}{\partial D_x}\frac{\partial D_x}{\partial p_x} = 0$$

Solving for $\dfrac{\partial D_x}{\partial p_x}$:

$$\frac{\partial D_x}{\partial p_x} = -\frac{\frac{\partial F}{\partial p_x}}{\frac{\partial F}{\partial D_x}}$$

We calculate the partial derivatives:

- $\dfrac{\partial F}{\partial p_x} = 1 - D_x e^{p_x p_y} p_y$
- $\dfrac{\partial F}{\partial D_x} = 1 - e^{p_x p_y}$

Thus:

$$\left.\frac{\partial D_x}{\partial p_x}\right|_{P_0} = -\frac{1 - D_{x_0} e^{p_{x_0} p_{y_0}} p_{y_0}}{1 - e^{p_{x_0} p_{y_0}}}$$

We observe that the result depends on the values of x, y, and D_x. If the derivative is positive, the good is classified as a Giffen good; if negative, it is a typical good.

2. First, find $\dfrac{\partial D_x}{\partial p_y}$:

$$\frac{\partial F}{\partial p_y} + \frac{\partial F}{\partial D_x}\frac{\partial D_x}{\partial p_y} = 0$$

Solving for $\dfrac{\partial D_x}{\partial p_y}$:

$$\frac{\partial D_x}{\partial p_y} = -\frac{\frac{\partial F}{\partial p_y}}{\frac{\partial F}{\partial D_x}}$$

We calculate the partial derivatives:

- $\dfrac{\partial F}{\partial p_y} = 1 - D_x e^{p_x p_y} p_x$
- $\dfrac{\partial F}{\partial D_x} = 1 - e^{p_x p_y}$

Thus:

$$\left.\frac{\partial D_x}{\partial p_y}\right|_{P_0} = -\frac{1 - D_{x_0} e^{p_{x_0} p_{y_0}} p_{x_0}}{1 - e^{p_{x_0} p_{y_0}}}$$

Now, compute the cross-price elasticity of demand with respect to p_y:

$$E_{D_x, p_y} = \left.\frac{\partial D_x}{\partial p_y} \cdot \frac{p_{y_0}}{D_{x_0}}\right|_{P_0}$$

Substituting:

$$E_{D_x,p_y} = \left(-\frac{1 - D_{x_0}e^{p_{x_0}p_{y_0}}p_{x_0}}{1 - e^{p_{x_0}p_{y_0}}}\right) \cdot \frac{p_{y_0}}{D_{x_0}}$$

The elasticity indicates that if the price of good y increases by 1%, the demand for good x will change by $\left(-\frac{1 - D_{x_0}e^{p_{x_0}p_{y_0}}p_{x_0}}{1 - e^{p_{x_0}p_{y_0}}}\right) \cdot \frac{p_{y_0}}{D_{x_0}}\%$. The variation depends on the sign of the result.